Maximize ALL Students’ Mathematical Learning through the Use of Powerful Instructional Strategies and Techniques

Webinar Series

Prepared for the Region 14 State Support Team

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IDENTIFYING AND USING ERROR PATTERNS TO INFORM INSTRUCTION FOR STUDENTS STRUGGLING IN MATHEMATICS WEBINAR

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Topics for Today

• Overall purpose is to improve student learning outcomes in mathematics through the use of more effective instructional techniques
• Error Analysis and Instructional Implications
  – Purpose of Conducting Error Analysis
  – Procedures to Conduct an Error Analysis
  – Using Error Analysis Results to inform Instruction
  – Micro-Instructional Progression
• Conclusion and Wrap-Up

Components of Effective Mathematics Programs

Mathematics Curriculum & Interventions
100% Math Proficiency
Teacher Content & Instructional Knowledge
Instruction Matters
Assessment & Data-Based Decisions

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Error Pattern Analyses

• As math problems become more complex, students need to go through a series of steps to solve problems.
• Often an error in any of these steps can cause failure in the final response.
• As a result, it is important to determine what the error is and provide targeted instructional remediation to correct the error.

Error Analysis Process

Error Analysis Process

- **STEP 1**: Collect and chart data.
- **STEP 2**: Analyze data.
- **STEP 3**: Establish shared expectations for implementing specific changes.
- **STEP 4**: Implement changes consistently.
- **STEP 5**: Collect, chart, and analyze post data.

Error Analysis

Purpose

**Error analysis**

- Involves reviewing the student’s independent work (e.g., seatwork, quizzes, tests, progress monitoring) to identify specific error types and patterns
- Helps to set priorities for teaching

**Error patterns**

- Indicate areas in need of further instruction
- Constitute a database for determining what content and strategies to teach

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Error Analysis Process

1. Identify errors on student’s independent work
2. Categorize errors by type
   - Conceptual
   - Factual
   - Procedural
   - Careless
3. Look for error patterns within each error type and across each error type
4. Provide instruction to remediate the error

Mathematical Error Types

1. Conceptual Errors
2. Factual Errors
   - AKA Computational Errors
3. Procedural Errors
4. Careless Errors

Errors occurring more than two times are considered a pattern and will require targeted instruction to correct.
Conceptual Errors

- Conceptual Errors
  - Students have developed misconceptions and have very poor understanding of concepts, procedures, and appropriate application.
  - Conceptual errors could also be related to vocabulary.
  - Difficult to differentiate between conceptual errors and procedural errors.
  - May require significant amounts of reteaching at the concrete and/or representational levels.

Types of Factual Errors

1. Sign Identification
2. Digit Identification
3. Counting errors
4. Computational Errors
5. Place Value Identification
6. Vocabulary Terminology
7. Incorrect formula use

Often possible to correct with minimal instructional time.
Types of Procedural Errors

- Equivalence
- Multiplying all Digits
- Regrouping
- Exponent Procedures
- Correct Alignment
- Equivalent fractions
- Decimal Position
- Solving simple equations

Requires **specific** and **targeted** remediation that matches the procedural error. These are **most common type** of error and will continue to occur across school years if not remediated.

**Practice is generally not effective for remediating procedural errors**

Getting Started with Identifying Error Patterns

- Analyze student’s work to determine a common mistake.
  - Review independent work from 3 different samples
- Some mistakes are “careless” but, many mistakes present a common error pattern.
- Very often, the student develops the error through a poorly applied algorithm or the mixing up of procedures.
- Error patterns in student work is more common than most teachers realize.
Case Study in Error Analysis

Tyler’s Error Analysis

1. Examine Student’s Independent Work
   - Basic Facts
   - Computation
     • Addition
     • Subtraction
     • Multiplication
     • Division

2. Identify specific error patterns (Error Analysis)

3. Develop a “Profile” of errors

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## Case Study in Error Analysis

**Data Recording sheet of Tyler’s Basic Fact Errors**

<table>
<thead>
<tr>
<th>FACTS</th>
<th>Addition</th>
<th>Subtraction</th>
<th>Multiplication</th>
<th>Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>Record all errors observed in student’s work</td>
<td>4+2=5</td>
<td>10+3=12</td>
<td>9+7=15</td>
<td>8+8=15</td>
</tr>
<tr>
<td></td>
<td>9+5=45 (x)</td>
<td>3+3=0 (-)</td>
<td>10+7=70 (x)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8+8=15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error Patterns</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Case Study in Error Analysis

Fluency and Automaticity Instructional Strategies

- Fluency of facts is vital, but instruction for conceptual understanding must occur first
- Fluency activities must be cumulative and **REGULAR**
  - Newly introduced facts receive intensive practice, while previously introduced facts receive less intensive, but still **SYSTEMATICALLY PLANNED**.
- Fluency building activities should **NOT** use up all of the allocated math time... **5-10 minutes**
- Fact **fluency instruction** is often **overlooked** by most math programs or provide ineffective practice opportunities
- **See webinar on Fluency and Automaticity**

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Procedural Errors: Addition

6 Separate Steps: (Procedural and Factual Errors)

1. **Addition Facts:** Are the single digit addition sums computed correctly?
2. **Regroup (Inside):** Are regroupings assigned to the proper column?
3. **Regroup (Outside):** Is the last regrouping part of the answer?
4. **Adding the Regroup:** Are regrouped numbers added with the proper column?
5. **Decimal:** Is the decimal in the correct place?
6. **Label:** Is the label in the correct place? (Dollar & cents sign).

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Procedural Errors: Addition

1. **Regroup-inside**
   - 4 6
   - 5 7
   - 10 3

2. **Addition facts**
   - Adding the regroup

3. **Regroup-outside**

---

Procedural Errors: Subtraction

4 Separate Steps:

1. **Direction of Subtraction:** Is subtraction always carried out in the proper direction?
2. **Subtraction Facts:** Are the single digit subtraction tasks computed correctly?
3. **Where to Regroup:** Does the student regroup from the correct column?
4. **Conversions:** After regrouping, does the student make the appropriate conversions in the adjacent columns?
**Procedural Errors: Subtraction**

![Diagram of subtraction process](image1)

1. Where to regroup
2. Conversions
3. Subtraction Facts
4. Direction of Subtraction

**Case Study in Error Analysis**

Review Tyler’s completed subtraction problem solutions and identify the error pattern in his work.

\[
\begin{array}{cccc}
1.1812 & 8.712 & 826 & 25.610 \\
-0.94 & -4.23 & -21 & -3.3 \\
1.98 & 4.59 & 805 & 25.37
\end{array}
\]

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Case Study in Error Analysis

Review Tyler’s completed subtraction problem solutions and identify the error pattern in his work.

\[
\begin{array}{cccc}
1.828 & 72826 & 826 & 25.610 \\
-94 & -4.23 & -21 & -33 \\
1.984 & 59 & 805 & 25.372
\end{array}
\]

Conversion Error

Case Study in Error Analysis

Tyler’s error pattern profile

Subtraction Error Pattern:

• Conversion Procedures:
  After regrouping, student does not properly reduce the digit regrouped from

Addition Error Pattern:

• Addition Fact:
  Student misses various addition facts by 1 (counting strategy???)
Case Study in Error Analysis

Summarize Error Patterns

• Facts:
  • Addition Facts (plus 1)
  • Multi. & Division >5
  • Sign Identification

• Subtraction Computation:
  • Conversion procedure

Addition Computation:
  • No major problems
  • Addition Facts (plus 1)

Procedural Errors: Multiplication

7 Separate Steps:

1. Multiplication Facts: Are the single digit multiplication processes completed accurately?

2. Multiplying All Combinations: Are all different combinations of multiplication attempted?

3. Regroup (Inside): Are regroups assigned to the proper column?

4. Regroup (Outside): Is the last regrouping part of the product?

5. Adding the Regroup: Are regrouped numbers combined with the proper column?

6. Lining up Addition: Are the intermediate products lined up correctly?

7. Addition: Is the final addition process carried out properly.
Procedural Errors: Multiplication

1. Correct Multipliers: Are the correct multipliers being chosen?
2. Multiplication: Regardless of what multipliers are being chosen, is the multiplication being carried out correctly?
3. Alignment: Is the result of the multiplication aligned in the correct position?
4. Subtraction: Is subtraction completed correctly?
5. Bringing Down: Does the student bring down the correct digit at the appropriate times?
6. Stopping: Is the procedure stopped at the appropriate time?
7. Fraction/Decimal: Are remainders dealt with appropriately?

Procedural Errors: Division

7 Separate Steps:

1. Correct Multipliers: Are the correct multipliers being chosen?
2. Multiplication: Regardless of what multipliers are being chosen, is the multiplication being carried out correctly?
3. Alignment: Is the result of the multiplication aligned in the correct position?
4. Subtraction: Is subtraction completed correctly?
5. Bringing Down: Does the student bring down the correct digit at the appropriate times?
6. Stopping: Is the procedure stopped at the appropriate time?
7. Fraction/Decimal: Are remainders dealt with appropriately?
Procedural Errors: Division

Correcting Student Errors

**Micro-Instruction**

- Targeting the specific error identified during the error analysis
  - Pinpoint instruction
- Teacher directed and controlled instruction and practice
- Focusing the student on the “place” in the process where the error occurred
- Very different than traditional reteaching the whole process
Implications for Instruction

- Use of effective teaching strategies (explicit instructional techniques from previous webinars)
  - Pinpoint error types and efficiently teach or reteach the skill
- Determine the instructional strategy to teach the skill.
- Be sure necessary preskills for strategies are present.
  - Do students understand big idea (i.e., concept)?
  - Do students know facts in a computation problem?

Tyler’s Error Pattern

Conversions in Subtraction with Regrouping

- Remind students of the rule relationship
  - If the bottom number is bigger than top number, you must regroup
  - More place value based language in terms of *subtracting more ones than are available*
- Teach steps of subtraction conversion
  - Subtract from column where regrouping
  - Convert the number from which you regrouped
  - Regroup number in appropriate column
  - Continue Subtraction

*Stein, Silbert, & Carnine, (1997)*
Instructional Progression

- 3 to 5 examples
  - Teacher directed and then guided practice
  - Teacher support is faded
- Conversions required in different place values in the problem
- Discrimination items (non-example)
  - Examples where conversions are not required
- **Focus instruction on the error pattern and NOT the entire problem process.**

Examples and Non-examples

Why?

- Establish the boundaries of rule
- Application of rules
- Under-generalization and overgeneralization
- Focus on the critical attributes(s)
- Textbooks do a poor job with non-examples in teacher demonstration

Archer & Hughes, 2011
Teaching Subtraction Conversions: Example Problems

<table>
<thead>
<tr>
<th>3 4</th>
<th>9 2 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>- 1 6</td>
<td>- 9 2</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td>9 7</td>
<td></td>
</tr>
<tr>
<td>- 2 3</td>
<td>5 2 2</td>
</tr>
<tr>
<td></td>
<td>- 3 0 8</td>
</tr>
</tbody>
</table>

Summary of Error Analysis

Error analysis
- Involves reviewing the student’s independent work (e.g., seatwork, homework, CBM probes) to identify specific error types and patterns
- Helps to set priorities for teaching

Error patterns
- Indicate areas in need of further instruction
- Constitute a database for determining what content and strategies to teach

Micro Instructional Remediation
- Specifically Targets are of error
- Focuses attention on specific error versus the entire problem solution
Classroom Implementation

- Teachers bring 3-4 samples of ON student’s work to a grade level planning meeting with a “semi-completed” error analysis.
- Each teacher presents the student’s work and the error analysis with special attention given to the error pattern.
- Discuss and develop a micro instruction mini lesson progression to implement for this student as well as other students with similar error patterns.

QUESTIONS?

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